Program and Book of Abstracts

Conference
The Cahn-Hilliard equation - recent advances and new challenges

Chęciny, 22 – 26.04.2024

Scientific committee :

Jose Antonio Carrillo, Piotr Gwiazda,
Jakub Skrzeczkowski, Zuzanna Szymańska

Organizing committee :

Katarzyna Kulczycka-Mierzejewska, Piotr Gwiazda,
Jakub Skrzeczkowski, Zuzanna Szymańska

Program of the Conference

Sunday, 21.04.2024

17:00	**Departure of the bus from Warsaw to Chęciny**
20:00	**Dinner**

Monday, 22.04.2023

9:00	**Breakfast**
9:55 - 10:00	**Opening of the conference**
10:00 - 10:40	**Maurizio Grasselli** *Nonlocal Cahn-Hilliard equations*
10:45 - 11:25	**Andrea Poiatti** *Strict separation property in phase-field models: where are we?*
11:30 - 12:00	**Coffee break**
12:00 - 12:40	**Carles Falcó** *Nonlocal models of cell-cell adhesion and their Cahn-Hilliard approximation*
12:45 - 13:05	**Giulia Cavalleri** *A phase field model of Cahn–Hilliard type for tumour growth with mechanical effects and damage*
13:30	**Lunch**
15:00 - 15:40	**Lara Trussardi** *Asymptotics and Optimal Control for a Cahn-Hilliard-Reaction-Diffusion model for tumor growth*
15:45 - 16:05	**Jonas Stange** *On a convective Cahn–Hilliard system with dynamic boundary conditions*
16:10 - 16:40	**Coffee break**
16:40 - 19:00	**Free afternoon for discussions**
19:00	**Dinner**

Tuesday, 23.04.2023

9:00	**Breakfast**
9:45 - 10:25	**Alain Miranville** *The Cahn-Hilliard equation with a source term*
10:30 - 11:10	**Charles Elbar** *Pressure jump in the Cahn-Hilliard equation*
11:15 - 11:40	**Coffee break**
11:40 - 12:25	**Julian Fischer** *A weak-strong uniqueness principle for De Giorgi type solutions to the Mullins-Sekerka equation*
12:30	**Lunch**
13:00 - 19:00	**Free afternoon (EXCURSION)**
19:00	**Dinner (BONFIRE)**

Wednesday, 24.04.2024

9:00	**Breakfast**
10:00 - 10:40	**Antonio Esposito** *Variational approach to fourth-order aggregation-diffusion PDEs*
10:45 - 11:25	**Alejandro Fernández-Jiménez** *Asymptotic behaviour for a family of Aggregation Diffusion equations*
11:30 - 12:00	**Coffee break**
12:00 - 12:40	**Sebastian Hensel (online)** *Weak solutions of Mullins-Sekerka flow as a Hilbert space gradient flow*
12:45 - 13:05	**Matteo Fornoni** *Maximal regularity and optimal control for a non-local Cahn-Hilliard tumour growth model*
13:30	**Lunch**
15:00 - 15:40	**Jakub Woźnicki** *Cahn–Hillard and Keller–Segel systems as high-friction limits of Euler–Korteweg and Euler–Poisson equations*
15:45 - 16:05	**Akash Parmar** *Fractional regularity for scalar conservation laws with discontinuous flux*
16:10 - 16:40	**Coffee break**
16:40 - 19:00	**Free afternoon for discussions**
19:00	**Dinner**

Thursday, 25.04.2024

9:00	**Breakfast**
10:00 - 10:40	**Ciprian Gal** *A general paradigm of binary phase-segregation processes through the lens of four critical mechanisms*
10:45 - 11:25	**Piotr Gwiazda** *TBA*
11:30 - 12:00	**Coffee break**
12:00 - 12:40	**Hangjie Ji** *Coarsening and mean field control of volatile droplets*
12:45 - 13:10	**Kalina Nec** *Mathematical modelling of hysteresis in the epithelial-mesenchymal transition*
13:30	**Lunch**
15:00 - 15:40	**Jakub Skrzeczkowski** *Several derivations of the degenerate Cahn-Hilliard equation via singular limits*
15:45 - 16:25	**Nilasis Chaudhuri** *Analysis for hydrodynamic model of swarming*
16:30 - 17:00	**Coffee break**
17:00 - 19:00	**Free afternoon for discussions**
19:00	**Dinner (CONFERENCE DINNER)**

Friday, 26.04.2024

8:00	**Breakfast**
9:00	**Departure from Chęciny to Warsaw**

A phase field model of Cahn–Hilliard type for tumour growth with mechanical effects and damage

Giulia Cavalleri

University of Pavia

Abstract

We introduce a new diffuse interface model for tumour growth in the presence of a nutrient, in which we take into account mechanical effects and reversible tissue damage. The highly nonlinear PDEs system mainly consists of a Cahn–Hilliard type equation that describes the phase separation process between healthy and tumour tissue. This equation is coupled to a parabolic reaction-diffusion equation for the nutrient and a hyperbolic equation for the balance of forces, including inertial and viscous effects. The main novelty is the introduction of cellular damage, whose evolution is ruled by a parabolic differential inclusion. We are able to prove a global-in-time existence result for weak solutions by passing to the limit in a time-discretized and regularised version of the system.

Analysis for hydrodynamic model of swarming

Nilasis Chaudhuri

University of Warsaw

Abstract

In this talk, we consider a one-dimensional hydrodynamic model featuring nonlocal attraction-repulsion interactions and singular velocity alignment. We introduce a two-velocity reformulation and the corresponding energy-type inequality, in the spirit of the Bresch-Desjardins estimate. We identify a dependence between the communication weight and interaction kernel, and between the pressure and viscosity term, allowing for this inequality to be uniform in time. It is then used to study the long-time asymptotics of solutions.

Pressure jump in the Cahn-Hilliard equation

Charles Elbar

Laboratoire Jacques Louis Lions, Sorbonne Université

Abstract

We model a tumor as an incompressible flow considering two antagonistic effects: repulsion of cells when the tumor grows (they push each other when they divide) and cell-cell adhesion which creates surface tension. To take into account these two effects, we use a 4th-order parabolic equation: the Cahn-Hilliard equation. The combination of these two effects creates a discontinuity at the boundary of the tumor that we call the pressure jump. To compute this pressure jump, we include an external force and consider stationary radial solutions of the Cahn-Hilliard equation. We also characterize completely the stationary solutions in the incompressible case, prove the incompressible limit and prove convergence of the parabolic problems to stationary states. I intend to review a number of results about nonlocal Cahn-Hilliard equations with constant or degenerate mobility. These results have been recently obtained by various authors. Coupling with hydrodynamics (i.e. Navier-Stokes system and Darcy's law) will also be discussed.

Variational approach to fourth-order aggregation-diffusion PDEs

Antonio Esposito

University of Oxford

Abstract

The seminar concerns the analysis of fourth-order aggregation-diffusion equations using an optimal transport approach. These models have been recently obtained as approximation of nonlocal systems of PDEs describing cell-cell adhesion, which is a crucial mechanisms regulating collective cell migration during tissue development, homeostasis and repair. In a recent work, we use the 2-Wasserstein gradient flow structure of such equations to give sharp conditions for global in time existence of weak solutions, in any dimension and for general initial data. The energy involved presents two competing effects: the Dirichlet energy and the power-law internal energy. Homogeneity of the functionals reveals critical regimes that we analyse. In addition, we study a system of two Cahn-Hilliard-type equations exhibiting an analogous gradient flow structure. This is based on a joint work with J. A. Carrillo, C. Falcó, and A. Fernández-Jiménez in Oxford.

Nonlocal models of cell-cell adhesion and their Cahn-Hilliard approximation

Carles Falcó

University of Oxford

Abstract

Cell-cell adhesion regulates cell migration during tissue development, homeostasis and repair, allowing cell populations to self-organise and form and maintain complex tissue shapes. Adhesive forces are highly linked to the cell geometry and often, continuum models represent these by nonlocal attractive interactions. In this talk, we will explain how such models can be approximated by Cahn-Hilliard type equations in the limit of short-range interactions. The resulting model is a local aggregation-diffusion equation, resembling a thin-film type equation, and numerical simulations in one and two dimensions reveal that it still shows the diversity of patterns observed both in experiments and in previously used nonlocal models. Moreover, this local equation can be written as gradient flow with respect to the 2-Wasserstein metric, which motivates the use of variational methods to prove existence. The existence results rely on a bound from below of the associated free energy functional, which is given by a suitable functional inequality. Finally, we will discuss generalisations of the existence theory to the case of systems of interacting populations.

Asymptotic behaviour for a family of Aggregation Diffusion equations

Alejandro Fernández-Jiménez

University of Oxford

Abstract

This talk is concerned with the asymptotic behaviour of two types of Aggregation-Diffusion PDEs. On the first part, I will study convergence to a stationary state solution for the Cahn-Hilliard equation

$$\partial_t \rho = -\operatorname{div}(\rho \nabla(\Delta \rho)) - \chi \Delta \rho^m, \tag{CH}$$

where $1 < m < m_c := 2 + 2/d$. The method that I will follow to obtain this result relies on the gradient flow structure of (CH).

In the second part of the talk, I will discuss about the asymptotic behaviour of the family of Aggregation-Diffusion equations

$$\partial_t \rho = \Delta \rho^m + \operatorname{div}(\rho \nabla(V + W * \rho)), \tag{ADE}$$

for $0 < m < 1$, and V, W regular enough potentials bounded from below. Using compactness arguments, the gradient flow structure of the problem, and viscosity solutions I am able to characterise the time limit of the solutions of (ADE) and discuss the existence of Dirac deltas.

This talk presents joint work with J.A. Carrillo, A. Esposito, D. Gómez-Castro, and J. Skrzeczkowski.

A weak-strong uniqueness principle for De Giorgi type solutions to the Mullins-Sekerka equation

Julian Fischer

Institute of Science and Technology Austria

Abstract

The Mullins-Sekerka equation describes volume-preserving phase separation and coarsening processes; it arises in particular as the sharp-interface limit of the Cahn-Hilliard equation with double-well potential. We establish a weak-strong uniqueness principle for De Giorgi type solutions to the two-phase Mullins-Sekerka equation in the planar case: As long as a classical solution to the evolution problem exists, any De Giorgi type weak solution as constructed by Hensel-Stinson must coincide with it. In particular, in the absence of geometric singularities their notion of weak solutions does not introduce a mechanism for (unphysical) nonuniqueness. We also derive a stability estimate with respect to changes in the data.

Joint work with Sebastian Hensel, Tim Laux, and Theresa Simon.

Maximal regularity and optimal control for a non-local Cahn-Hilliard tumour growth model

Matteo Fornoni

Università degli Studi di Pavia

Abstract

We consider a non-local tumour growth model of phase-field type, describing the evolution of tumour cells through proliferation in the presence of a nutrient. The model consists of a coupled system, incorporating a non-local Cahn-Hilliard equation for the tumour phase variable and a reaction-diffusion equation for the nutrient, and also including chemotaxis mechanisms in the form of cross-diffusion terms. We establish novel regularity results for the solutions of this model, by applying maximal regularity theory in weighted L^p spaces. Such a technique enables us to prove the existence and uniqueness of a highly regular solution, which is the foundation for addressing an optimal distributed control problem. Indeed, we aim to identify a suitable therapy, capable of guiding the evolution of the tumour towards a predefined target. Specifically, we prove the existence of an optimal therapy and then, by studying the Fréchet-differentiability of the control-to-state operator and introducing the adjoint system, we derive first-order necessary optimality conditions.

A general paradigm of binary phase-segregation processes through the lens of four critical mechanisms

Ciprian Gal

Florida International University

Abstract

We formulate and investigate a nonlocal nonlinear parabolic equation that characterizes phase-segregation phenomena in binary systems, influenced by four distinct mechanisms. These mechanisms offer full user control over both the temporal and spatial scales of the phase separation process within a bounded domain. We discuss minimal conditions for well-posedness and examine solution regularity across various scenarios, encompassing both smooth and singular entropy density potentials. We will share some preliminary numerical simulations and physical experiments, which elucidates the intricate interplay of the four-point physical mechanism underlying the phase segregation process.

Nonlocal Cahn-Hilliard equations

Maurizio Grasselli

Politecnico di Milano

Abstract

I intend to review a number of results about nonlocal Cahn-Hilliard equations with constant or degenerate mobility. These results have been recently obtained by various authors. Coupling with hydrodynamics (i.e. Navier-Stokes system and Darcy's law) will also be discussed.

Weak solutions of Mullins-Sekerka flow as a Hilbert space gradient flow

Sebastian Hensel

University of Bonn

Abstract

Weak solution theories are in general necessary for interface evolution problems as topology changes naturally occur. If the topology change is realized through a physically unstable singularity, this results in non-uniqueness of solutions afterward. The best one can thus expect is a weak-strong uniqueness principle; and this was proven in recent years for prominent examples (e.g., multiphase mean cur- vature flow). At the level of a weak solution concept, the key conceptual ingredient for these results is given by the dissipative nature of the problems.

With this in mind, I will motivate and describe a novel weak solution theory for the sharp interface limit of the Cahn–Hilliard equation with double-well potential: the Mullins–Sekerka equation. This solution theory is essentially only encoded in terms of a single sharp energy dissipation principle, taking direct inspiration from De Giorgi's approach to gradient flows or the Sandier–Serfaty approach to evolutionary Γ-convergence, and it is the first for the Mullins–Sekerka equation allowing for a subsequent study of uniqueness properties.

This is joint work with Kerrek Stinson (Arch. Ration. Mech. Anal. 248, 8, 2024).

Coarsening and mean field control of volatile droplets

Hangjie Ji

North Carolina State University

Abstract

A lubrication model can be used to describe the dynamics of a weakly volatile viscous fluid layer on a hydrophobic substrate. Thin layers of the fluid are unstable to perturbations and break up into slowly evolving interacting droplets. In this talk, we will first present a reduced-order dynamical system derived from the lubrication model based on the nearest-neighbour droplet interactions in the weak condensation limit. Dynamics for periodic arrays of identical drops and pairwise droplet interactions are investigated which provide insights to the coarsening dynamics of a large droplet system. Weak condensation is shown to be a singular perturbation, fundamentally changing the long-time coarsening dynamics for the droplets and the overall mass of the fluid in two additional regimes of long-time dynamics. For the second part of the talk, I will briefly discuss our recent results on a mean field control formulation for droplet dynamics. Numerical examples with high-order finite element computations for droplet formation, transport, merging, and splitting demonstrate the effectiveness of the proposed mean field control. This talk is based on joint works with Thomas Witelski, Guosheng Fu, and Wuchen Li.

The Cahn-Hilliard equation with a source term

Alain Miranville

Université de Poitiers

Abstract

Our aim in this talk is to study the Cahn-Hilliard equation with a (possibly nonlinear) source term. This has applications in tumor growth, image processing, population dynamics, ... In particular, we prove, for a logarithmic potential, the existence of a global in time solution.

Mathematical modelling of hysteresis in the epithelial-mesenchymal transition

Kalina Nec

University of Warsaw

Abstract

Cancer progression is facilitated by epithelial-mesenchymal transition (EMT), which transforms epithelial cells into mesenchymal cells. A computational model is being developed to explain the hysteresis effects caused by varying concentrations of Transforming Growth Factor Beta (TGF-β). The model is based on ordinary differential equations and simulates the interactions between key molecular players like ZEB1/2, MicroRNA-200, and E-cadherin using Python and numerical libraries like NumPy, SciPy and Matplotlib. We investigate the effects of short versus prolonged (TGF-β) exposures on the reversibility of EMT, with a focus on potential therapeutic interventions that take advantage of the hysteretic response of the system. Additionally, the analysis discusses the mathematical underpinnings, such as sigmoid functions and the use of a distributed delay kernel in order to better represent biological delays. It appears that therapeutic timing and duration might be crucial in controlling cancer progression, indicating future directions for improving model accuracy and clinical applicability. EMT is a dynamic, reversible process. This presentation aims to provide insights into strategies to treat cancer effectively.

Nonlocal Cahn-Hilliard equations

Akash Parmar

University of Warsaw

Abstract

In this talk, we discuss the regularity aspects of the entropy solutions for scalar conservation laws with discontinuous flux. From the work [Adimurthi et al., Comm. Pure Appl. Math. 2011], it is well-known that there exists initial data in BV such that the corresponding entropy solution does not belong to BV space. Consequently, we investigate the necessity of fractional BV^s spaces, which is a bigger space than BV, with $0 < s \leq 1$. We prove the optimal regularizing effect for the discontinuous flux with L^f initial data. The optimal regularizing effect in BV^s is proven in an important case using control theory, and the fractional exponent s is at most $1/2$, even when the fluxes are uniformly convex.

Strict separation property in phase-field models: where are we?

Andrea Poiatti

University of Vienna

Abstract

In phase-field models, like Cahn-Hilliard or Allen-Cahn equations with the adoption of singular potential, it is of paramount importance to assess that the solution stays uniformly away from the pure phases, instantaneously or at least eventually in time. This is useful to study higher-order regularization properties of the solution, as well as its longtime behavior. In this talk I would like to present some recent results, concerning the validity of this property in many phase-field models, from the Cahn-Hilliard equation (local and nonlocal) to the conserved Allen-Cahn equation, either for binary fluids or for multi-component mixtures. I will try to outline the most updated state of the art about this validity of the strict separation property, hopefully giving also some possible ideas towards the solution of some still open questions related to the topic.

Several derivations of the degenerate Cahn-Hilliard equation via singular limits

Jakub Skrzeczkowski

University of Oxford

Abstract

The degenerate Cahn-Hilliard equation, originally introduced in material science, is nowadays used in several different fields, including biology (tumor growth, cell-cell adhesion) and fluid dynamics. In this talk, we discuss several derivations of this equation: via hydrodynamic limit from the Vlasov equation (arXiv:2208.01026 - published in CIMP; arXiv: 2306.06486), from the nonlocal equation and related interacting particle system in the spirit of Giacomin-Lebowitz work (arXiv:2208.08955 - published in JDE; arXiv:2303.11929), and most recently, from the Euler-Korteweg equation (arXiv:2305.01348). In most of these cases, fully rigorous solutions are available only in the case of torus and so, we will stress fundamental difficulties related to the presence of physical boundaries. This is a joint work with C. Elbar, B. Perthame (Paris), J. A. Carrillo (Oxford), M. Mason (Milan), A. Świerczewska-Gwiazda, and P. Gwiazda (Warsaw).

On a convective Cahn–Hilliard system with dynamic boundary conditions

Jonas Stange

Universität Regensburg

Abstract

We consider a general class of convective Cahn–Hilliard systems with dynamic boundary conditions. In contrast to classical Neumann boundary conditions, the dynamic boundary conditions of Cahn–Hilliard type allow for dynamic changes of the contact angle between the diffuse interface and the boundary, a convection-induced motion of the contact line as well as absorption of material by the boundary. The coupling conditions for bulk and surface quantities involve parameters $K, L \in [0, \infty]$, whose choice declares whether these conditions are of Dirichlet, Robin or Neumann type.

In this talk, I present some recent results on the well-posedness of this system. For regular potentials, the existence of weak solutions in the case $K, L \in (0, \infty)$ are proven by means of a Faedo–Galerkin approach, whereas for all other cases, the existence is shown by means of the asymptotic limits, where K and L are sent to zero or to infinity, respectively. Eventually, we establish higher regularity for the phase-fields, and we prove the uniqueness of weak solutions under additional assumptions on the mobility functions. Finally, we prove analogous results in the case of singular potentials. For the analysis in this case, we regularise singular potentials by a Yosida approximation, which allows us to apply the results for regular potentials, and eventually pass to the limit in this approximation scheme.

This is based on joint work with Patrik Knopf (Universität Regensburg).

Asymptotics and Optimal Control for a Cahn-Hilliard-Reaction-Diffusion model for tumor growth

Lara Trussardi

University of Graz

Abstract

We study nonlocal-to-local asymptotics for a tumor-growth model coupling a viscous Cahn-Hilliard equation describing the tumor proportion with a reaction-diffusion equation for the nutrient phase parameter. First, we prove that solutions to the nonlocal Cahn-Hilliard system converge, as the nonlocality parameter tends to zero, to solutions to its local counterpart. Second, we provide first-order optimality conditions for an optimal control problem on the local model, accounting also for chemotaxis, and both for regular or singular potentials, without any additional regularity assumptions on the solution operator.

This is a joint work with Elisa Davoli, Elisabetta Rocca and Luca Scarpa.

Cahn–Hillard and Keller–Segel systems as high-friction limits of Euler–Korteweg and Euler–Poisson equations

Jakub Woźnicki

University of Warsaw

Abstract

We consider a singular, high-friction limit from Euler - Korteweg equations to the Cahn - Hillard system, with exponential pressure (exponent $\gamma > 1$). We will discuss the ideas of the solutions for both systems, and how one can compare both of them in spite of a different structure. We will also look at different technical issues appearing in the proof, which reduce the possibilities of the regularity of the pressure, and compare those with similar results in the field.

Our discussion will be joined with the consideration of the singular, high-friction limit from Euler—Poisson to the Keller—Siegel system, and we shall look at the problems that appear in this case as well.